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Short and Long Distance Interplay in Inclusive $B \rightarrow X_d \gamma$ Decay

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Abstract

We analyze the short and long distance contributions to inclusive $B \rightarrow X_d \gamma$ decay, paying particular attention to the dependence on the Cabibbo-Kobayashi-Maskawa parameter V_{td} . We discuss penguin diagrams with internal u and c quarks in the framework of the effective field theory. We also estimate the size of possible long range contributions by using vector meson dominance.

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1 Introduction

The radiative penguin decays have recently received considerable theoretical attention. The short distance QCD corrections to $b \rightarrow s \gamma$ decay have been calculated completely at the leading order [1] and partially at the next-to-leading order [2]; the branching ratio is in agreement (within the errors) with the experimental value from CLEO [3]. The penguin diagrams for the $b \rightarrow s \gamma$ decay are dominated by the virtual t -quark contribution that is proportional to the element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix V_{ts} . On the other hand, the $b \rightarrow d \gamma$ decay can, if dominated by the t -quark loop, provide information on V_{td} . This decay has not been detected yet. The expected branching ratio is approximately 10^{-5} (see e. g. [4]) and the statistics required is of order $10^7 - 10^8$ B mesons, which is within the reach of future CLEO/B factories. The principal experimental challenge is to keep a good control of the main background due to the $b \rightarrow s \gamma$ decay.

In this paper we will analyze the inclusive $B \rightarrow X_d \gamma$ decay. It has been suggested (see e.g. [5]) that its amplitude may not be proportional to $|V_{td}|$ because of sizable contributions from penguin diagrams with internal c and u quarks. We compare the relative contributions of penguin diagrams with heavy and light loops in the framework of the effective field theory. We are interested whether the dependence on the light masses is logarithmic or powerlike, since we expect that terms with powerlike dependence are negligible. We also aim to estimate possible long range contributions to the decay rate of inclusive $B \rightarrow X_d \gamma$; we adopt the phenomenological approach of vector meson dominance (VMD).

2 The Short Distance

The amplitude A for the $b \rightarrow s \gamma$ or $b \rightarrow d \gamma$ decay can be written as $A = v_t A_t + v_u A_u + v_c A_c$, where $v_t = V_{tb} V_{tx}^*$, $v_c = V_{cb} V_{cx}^*$, $v_u = V_{ub} V_{ux}^*$, with $x = s$ or d respectively. By using the unitarity of the CKM matrix, we can rewrite the amplitude in the form

$$A = v_t(A_t - A_c) + v_u(A_u - A_c). \quad (1)$$

In the case of $b \rightarrow s \gamma$, we have $v_t \sim O(\lambda^2)$ and $v_u \sim O(\lambda^5)$, where $\lambda \simeq 0.22$ is the Cabibbo suppression factor; thus, $v_u \ll v_t$ and it is generally considered safe to ignore the second term in (1). For the $b \rightarrow d \gamma$, however, v_t and v_u are comparable $\sim O(\lambda^3)$; that prompts us to examine more carefully the size of $A_u - A_c$.

Let us first consider the $b \rightarrow d \gamma$ amplitude in the absence of QCD corrections. The gauge invariant form for $A_{u,c,t}$ resulting from the expansion to the second order in the

external momenta is

$$A_q \equiv A_q^P + A_q^M \quad q = \{u, c, t\}, \quad (2)$$

with

$$\begin{aligned} A_q^P &\equiv F_P(m_q^2/m_W^2) \bar{d}(\hat{k}k^\mu - k^2\gamma^\mu)(1 - \gamma_5)b\epsilon_\mu^\gamma \\ A_q^M &\equiv F_M(m_q^2/m_W^2) \bar{d}\sigma^{\mu\nu}k_\nu[m_d(1 - \gamma_5) + m_b(1 + \gamma_5)]b\epsilon_\mu^\gamma, \end{aligned} \quad (3)$$

where ϵ_μ^γ is the photon polarization vector, k is the photon momentum and the coefficients F_P , F_M are independent of the momenta. A_q^P and A_q^M will be referred as the penguin and magnetic moment amplitudes. As worked out explicitly in [6], the dependence of A_q^M on the ratio m_q^2/m_W^2 is powerlike, while in A_q^P there are also logarithmic terms. This can be qualitatively understood in the following way. By naive power counting in the euclidean space, we observe that, before the expansion in the external momenta, the one loop diagrams have no IR divergence because of the massive W propagator. After the expansion, the only source of infrared (IR) divergence in the one loop diagrams is a logarithmic dependence on the internal quark masses, when the masses go to zero. The expansion induces the following IR behaviour, expressed symbolically in terms of the internal momentum l

$$A_q^P \sim k^2 \int d^2l \frac{1}{l^2}, \quad A_q^M \sim k_\nu \int d^3l \frac{1}{l^2}. \quad (4)$$

Therefore we expect IR divergence (and consequently logarithms in the light masses) only in the penguin terms². When the photon is real, only the amplitudes A_q^M contribute to the decay. On the other hand, A_c^M and A_u^M are strongly suppressed due to their powerlike dependence on m_q^2/m_W^2 . By using the results of Inami and Lim [6], we find

$$\frac{A_c - A_u}{A_t - A_c} = \frac{A_c^M - A_u^M}{A_t^M - A_c^M} \simeq 5 \times 10^{-4} \quad (m_t = 174 \text{ GeV}, m_c = 1.5 \text{ GeV}). \quad (5)$$

Therefore, in the absence of QCD corrections, it is justified to assume proportionality to v_t in the $b \rightarrow d \gamma$ decay.

In order to include QCD corrections, we introduce the effective field theory formalism and work at the lowest order in the weak interactions. The basis of operators $\{Q_1 \dots Q_8\}$ for the $b \rightarrow s \gamma$ decay is well known and we will not explicitly reproduce it here; the reader is referred to [1]. After the substitution of the s -quark with the d -quark, all the

²Note that this is not necessarily true in diagrams with more than one loop, since the presence of more internal loop momenta and of IR subdivergences alter the naive power counting of (4).

operators in this basis become suitable for the $b \rightarrow d \gamma$ decay. In addition, we have two new current-current operators generated by integrating out the W -boson

$$\begin{aligned} Q_1^u &= \bar{d}_\alpha \gamma_\mu (1 - \gamma_5) u_\beta \bar{u}_\beta \gamma_\mu (1 - \gamma_5) b_\alpha \\ Q_2^u &= \bar{d}_\alpha \gamma_\mu (1 - \gamma_5) u_\alpha \bar{u}_\beta \gamma_\mu (1 - \gamma_5) b_\beta. \end{aligned} \quad (6)$$

Since the effective theory and the full theory share, by definition, the same low energy behaviour, all possible logarithmic singularity in the light masses (for $m_c, m_u \rightarrow 0$) are cancelled at the matching. Therefore, at the matching scale $\mu = M_W$, the coefficients of the effective hamiltonian may have a powerlike and logarithmic dependence on the heavy masses, but only a powerlike dependence on the light masses. The only non-zero coefficients are

$$\begin{aligned} C_2(m_W) &= v_c \\ C_7(m_W) &= v_t \left[\frac{1}{2} F_2 \left(\frac{m_t^2}{m_W^2} \right) - O \left(\frac{m_c^2}{m_W^2} \right) \right] + v_u \left[O \left(\frac{m_u^2}{m_W^2} \right) - O \left(\frac{m_c^2}{m_W^2} \right) \right] \\ &\rightarrow v_t \frac{1}{2} F_2 \left(\frac{m_t^2}{m_W^2} \right) \\ C_8(m_W) &= v_t \left[\frac{1}{2} \tilde{F}_2 \left(\frac{m_t^2}{m_W^2} \right) - O \left(\frac{m_c^2}{m_W^2} \right) \right] + v_u \left[O \left(\frac{m_u^2}{m_W^2} \right) - O \left(\frac{m_c^2}{m_W^2} \right) \right] \\ &\rightarrow v_t \frac{1}{2} \tilde{F}_2 \left(\frac{m_t^2}{m_W^2} \right) \\ C_2^u(m_W) &= v_u \end{aligned} \quad (7)$$

where F_2 and \tilde{F}_2 are Inami-Lim coefficients³. The QCD rescaling does not change the powerlike dependence of the amplitude on the light masses. The anomalous dimension matrix for $b \rightarrow d \gamma$ differs from the anomalous dimension matrix for $b \rightarrow s \gamma$ only for the additional entries due to Q_1^u and Q_2^u . Thanks to their similarity with Q_1 and Q_2 , Q_1^u and Q_2^u mix with all the other operators in the same way. Due to the flavor difference, they do not mix with Q_1 and Q_2 . In $b \rightarrow s \gamma$ and $b \rightarrow d \gamma$, the amplitude is proportional to the so-called effective coefficient C_7^{eff} (see e.g. [7]), that includes finite contributions from matrix elements. At the leading order and in naive dimensional regularization, the only non-zero one-loop matrix elements are the matrix elements of Q_3, Q_4, Q_5, Q_6 with a massive internal b -quark. These finite terms have been calculated in the $b \rightarrow s \gamma$ case

³ F_2 is given by Eq. (B.3) in [6]. \tilde{F}_2 comes from the diagrams (a), (b), (c) and (d) in [6] when the Z is replaced by a gluon; therefore $F_2 = Q\tilde{F}_2 + \dots$

and are obviously left invariant by the inclusion of Q_1^u and Q_2^u in the basis. There are no other possible sources of logarithms in the light masses at the leading order; as a result, C_7^{eff} is

$$C_7^{eff}(\mu) = \eta^{16/23} C_7(m_W) + \frac{8}{3} (\eta^{14/23} - \eta^{16/23}) C_8(m_W) + (C_2(m_W) + C_2^u(m_W)) \sum_1^8 h_i \eta^{a_i}, \quad (8)$$

where $\eta = \alpha_s(m_W)/\alpha_s(\mu)$. The numbers a_i and h_i are given by the eigenvectors and eigenvalues of the anomalous dimension matrix [1]. By equation (7) and using the unitarity of the CKM matrix, we can see that the expression is still proportional to v_t , as pointed out also in [8].

It must be stressed that this result is peculiar to $b \rightarrow d \gamma$ (and to $b \rightarrow s \gamma$). For instance, in decays like $b \rightarrow (d, s) q \bar{q}$ there are non-zero penguin diagrams with a virtual photon and massless internal quarks. They have a logarithmic dependence on the light masses and introduce non-negligible contributions proportional to v_u in the amplitude [9].

At the next-to-leading order the situation is different. Now the matching is performed at α_s order with two loops diagrams. The expansion in the external momenta of the two loops diagrams gives rise to penguin and magnetic moment amplitudes like in (2) and (3). Since the naive power counting in (4) does not necessarily work at two loops, both amplitudes may have a logarithmic dependence on the internal masses. We expect the singularity in the light masses to be reproduced in the effective theory by the matrix elements of the four quark operators. This is similar to what happens in $b \rightarrow s e^+ e^-$; the logarithms cancelled at the matching in the coefficient of the operator $\bar{s} \gamma_\mu (1 - \gamma_5) b \bar{e} \gamma^\mu e$ are recovered at lower energy in the matrix elements of H_{eff} [10]. At the same way, in $b \rightarrow d \gamma$, nothing prevents the two loop matrix elements of Q_1, Q_2, Q_1^u, Q_2^u (proportional to v_c and v_u) to have a non-negligible logarithmic dependence on the light masses. Unfortunately, the next-to-leading calculation has not been completed even for the $b \rightarrow s \gamma$ decay, making a reliable estimate difficult; roughly speaking, we expect non-negligible terms proportional to v_u in (1) to be suppressed by an extra power of α_s . Since $\alpha_s(m_b) \sim 0.21$, however, this contribution may well be sizable.

In the inclusive $B \rightarrow X_d \gamma$ decay one has to take into account also the gluon bremsstrahlung corrections via the $b \rightarrow d + g + \gamma$ decay. These corrections have been partially calculated in [4]. The matrix elements of the four fermion operators in $b \rightarrow d + g + \gamma$ introduce additional terms that are not proportional to V_{td} with a logarithmic dependence on the light masses.

3 The Long Distance

The short distance approach leaves out possible contributions due to intermediate hadronic states. These contributions may be estimated by using the vector meson dominance (VMD) hypothesis, as suggested by [11]. According to VMD, the $b \rightarrow d \gamma$ decay is described by the $b \rightarrow d V$ decay, where V is a vector meson, (ψ and its excited states, ρ and ω), followed by the conversion $V \rightarrow \gamma$. Since the applicability itself of VMD is not well established in the case of radiative penguin decays, we will limit ourselves to give an order of magnitude of this contribution and we will make very simple hypothesis. By Lorentz invariance, the most general interaction proportional to the vector meson polarization ϵ_μ^V is given by

$$\begin{aligned} A \propto & \epsilon_\mu^V \bar{d} \{ \gamma^\mu [a_1 (1 - \gamma_5) + b_1 (1 + \gamma_5)] \\ & + \sigma^{\mu\nu} [a_2 (1 - \gamma_5) + b_2 (1 + \gamma_5)] p_\nu + \sigma^{\mu\nu} [a_3 (1 - \gamma_5) + b_3 (1 + \gamma_5)] q_\nu \\ & + [a_4 (1 - \gamma_5) + b_4 (1 + \gamma_5)] p^\mu + [a_5 (1 - \gamma_5) + b_5 (1 + \gamma_5)] q^\mu \} b, \end{aligned} \quad (9)$$

where $p_\mu \equiv p_\mu^d + p_\mu^b$ is the sum of d and b momenta and $q_\mu \equiv p_\mu^d - p_\mu^b$ is the vector meson moment. Terms proportional to $\bar{s} q^\mu (1 \pm \gamma_5) b$ are zero since the vector meson is on-shell and $\epsilon_\mu^V q^\mu = 0$. Since the quarks are on shell, we may use the Gordon decomposition to write

$$\begin{aligned} A \propto & \epsilon_\mu^V \bar{d} \{ \sigma^{\mu\nu} [a_2 (1 - \gamma_5) + b_2 (1 + \gamma_5)] p_\nu + \sigma^{\mu\nu} [a_3 (1 - \gamma_5) + b_3 (1 + \gamma_5)] q_\nu \\ & + [a_4 (1 - \gamma_5) + b_4 (1 + \gamma_5)] p^\mu \} b. \end{aligned} \quad (10)$$

The gauge invariance requires that only transverse terms couple to the photon; therefore, we obtain the following amplitude

$$A_T(b \rightarrow d V) = \frac{G_F}{\sqrt{2}} v_q f_V (m_V^2) \frac{m_V}{m_b} \left[a_3 \bar{d} \sigma^{\mu\nu} (1 - \gamma_5) q_\nu b + b_3 \bar{d} \sigma^{\mu\nu} (1 + \gamma_5) q_\nu b \right] \epsilon_\mu^V, \quad (11)$$

where f_V is the V decay constant (see Table 1) and e_V is the charge factor of the constituent states according to the quark model ($e_\rho = 1/\sqrt{2}$, $e_\omega = 1/(3\sqrt{2})$, $e_\psi = 2/3$). The two terms in (11) do not interfere ($m_d \simeq 0$) and give the same contribution to the transverse rate: $|A_T(b \rightarrow s \gamma)|^2 \propto \xi_V^2$ where $\xi_V \equiv \sqrt{|a_3|^2 + |b_3|^2}$.

Using the measured transverse decay rate for $B \rightarrow \psi X$, we can fit ξ_ψ . Experimentally [12, 13]

$$Br(B \rightarrow \psi + X) = 0.81 \pm 0.08\% \quad (12)$$

$$\Gamma_L/\Gamma = 0.78 \pm 0.17 \quad (1.4 \text{ GeV} < p_\psi < 2.0 \text{ GeV}), \quad (13)$$

$$\Gamma_L/\Gamma = 0.59 \pm 0.15 \quad (p_\psi < 2.0 \text{ GeV}), \quad (14)$$

where Γ_L is the longitudinal decay rate. The branching ratio (12) is for direct production of ψ , while the polarization data (14) and (13) include decay products of heavier charmoniums. It is known [13] that the momentum range $1.4 \text{ GeV} < p_\psi < 2.0 \text{ GeV}$ is dominated by the exclusive modes $\psi K(K^*)$, while the low momentum region $p_\psi < 1.1 \text{ GeV}$ is mostly from heavier charmoniums. We expect (13) to be an overestimate since ψK is purely longitudinal by kinematics, and (14) to be an underestimate since it is diluted by decay products. Here, we simply take the average and use $\Gamma_L/\Gamma = 0.7$ for direct inclusive ψ production. Using Eqs (11), (12) and $\Gamma_T/\Gamma = 1 - \Gamma_L/\Gamma = 0.3$ we obtain $\xi_\psi \simeq 0.19$.

The measured branching ratio for the inclusive B decay into ψ' is $\text{Br}(B \rightarrow \psi' + X) = (0.34 \pm 0.04 \pm 0.03) \%$ [13]. We assume that the measured branching ratio is equal to the direct branching ratio, since there is no known cascade process for ψ' production. By using again $\Gamma_T/\Gamma = 0.3$, we estimate $\xi_{\psi'} \simeq 0.18$. Experimental data are not available yet for the other ψ resonances, ρ and ω . Encouraged by ξ_ψ and $\xi_{\psi'}$ above, we assume the same a_3, b_3 and take $\xi_V = \xi_\psi = \sqrt{|a_3|^2 + |b_3|^2} \simeq 0.2$ for all vector mesons $V = \psi, \dots, \rho, \omega$. Then the VMD amplitude for the charmonium states is

$$A_T^\psi(b \rightarrow d\gamma) = e \frac{G_F}{\sqrt{2}} v_c e_\psi \sum_i \left(\frac{f_{\psi_i}^2(0)}{m_b} \right) \left[a_3 \bar{d} \sigma^{\mu\nu} (1 - \gamma_5) q_\nu b + b_3 \bar{d} \sigma^{\mu\nu} (1 + \gamma_5) q_\nu b \right] \epsilon_\mu^\gamma. \quad (15)$$

Note that now the decay constants are extended to $q^2 \rightarrow 0$. That gives rise to a suppression factor $f_{\psi_i}^2(0) = k f_{\psi_i}^2(m_{\psi_i}^2)$ where we take $k \simeq 0.12$ from Ref. [11, 14]; no suppression factor is taken for ρ and ω , whose masses are smaller. By substituting in (15) $v_c \rightarrow v_u$, $e_\psi \rightarrow e_{\rho(\omega)} 1/\sqrt{2}$, $f_{\psi_i}^2(m_{\psi_i}^2) \rightarrow f_{\rho(\omega)}^2(m_{\rho}^2)$, $k \rightarrow 1$ we obtain $A_T^{\rho(\omega)}(b \rightarrow d\gamma)$.

The contributions to the amplitude coming from ψ and its resonances are approximately equal to the contributions coming from ρ and ω intermediate states times the CKM factors

$$\frac{|A_T^\rho + A_T^\omega|}{|A_T^\psi|} \simeq 1.06 \frac{|v_u|}{|v_c|} \simeq 0.4. \quad (16)$$

The sign between the ψ mode and the $\omega + \rho$ mode may not be reliable, since, at least in the exclusive case ψK^* , we expect sizable final state interactions. However, if we ignore possible final state phases, the contribution proportional to v_u coming from charmonium states (by unitarity $v_c = -v_t - v_u$) approximately cancel the contribution coming from ρ and ω [11].

The short distance amplitude at the leading order in QCD corrections is

$$A^{SD}(b \rightarrow d\gamma) = e \frac{G_F}{\sqrt{2}} v_t C_7^{eff}(m_b) \frac{1}{4\pi^2} m_b \bar{d} \sigma^{\mu\nu} (1 + \gamma_5) q_\nu b \epsilon_\mu^\gamma, \quad (17)$$

where $C_7^{eff}(m_b) \simeq 0.3$ from Eqs (7) and (8).

The decay rate from (15) alone is about 10^{-3} of the short distance decay rate, in agreement with [11]. The change to decay rate from short distance due to long distance contributions proportional to v_u is less than 6% with respect to the short distance decay rate, while the change due to long distance contributions proportional to v_u is less than $12\% \times |v_u|/|v_t|$. The change in the decay rate due to $A_T^{\rho} + A_T^{\omega}$ is less than $6\% \times |v_u|/|v_t|$. Therefore, the ratio between the short and the long distance pieces is small with respect to the theoretical errors in the short distance decay rate itself which has been estimated to be about 25% [7].

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References

- [1] B. Grinstein, R. Springer, M. B. Wise, Nucl. Phys. B 339 (1990) 269; R. Grigjanis, P.J. O' Donnell, M. Sutherland, H. Navelet, Phys. Lett. B 213 (1988) 355; Phys. Lett. B (1992) 413E; M. Misiak, Nucl. Phys. B 393 (1993) 23; B439 (1995) 461E; K. Adel and Y.P. Yao, Mod. Phys. Lett. A 8 (1993) 1679; Phys. Rev. D 49 (1994) 4945. G. Cella, G. Curci, G. Ricciardi, A. Vicer , Phys. Lett. B 248 (1990) 181; Phys. Lett. B 258 (1991) 212; Phys. Lett. B 325 (1994) 227; Nucl. Phys. B 431 (1994) 417. M. Ciuchini, E. Franco, G. Martinelli, L. Reina and L. Silvestrini, Phys. Lett. B 316 (1993) 127; M. Ciuchini, E. Franco, L. Reina and L. Silvestrini, Nucl. Phys. B 421 (1994) 64.
- [2] A. Ali and C. Greub, Z. Phys. C 49 (1991) 431; Phys. Lett. B 259 (1991) 182; K. Adel and Y.P. Yao, UM-TH-93-20; M. Misiak and M. Munz, Phys. Lett. B 344 (1995) 308.
- [3] R. Ammar et al. (CLEO coll.), Phys. Rev. Lett. **71**, 674 (1993); B. Barish et al. (CLEO coll.), CLEO CONF 94-1; M. Athanas et al. (CLEO coll.), CLEO CONF 94-2.
- [4] A. Ali and C. Greub, Phys. Lett. B 287 (1992) 191.

- [5] D. Atwood, B. Blok, A. Soni, SLAC-PUB-95-6635.
- [6] T. Inami and C. S. Lim Progr. Theor. Phys. 65 (1981) 297; 1772E
- [7] A. J. Buras, M. Misiak, M. Munz, S. Pokorski, Nucl. Phys. B 424 (1994) 374.
- [8] A. Ali, V. M. Braun, H. Simma, Z. Phys. C 63 (1994) 437.
- [9] A. J. Buras and R. Fleischer, Phys. Lett. B 341 (1995) 379.
- [10] B. Grinstein, M.J. Savage, M. B. Wise, Nucl. Phys. B 319 (1989) 271.
- [11] N. G. Deshpande, X. He and J. Trampetic, OITS-564-REV (1995).
- [12] T. Browder, K. Honscheid and S. Playfer, in *B Decays*, edited by S. Stone, Second Edition, World Scientific, 1994.
- [13] R. Balest et al. (CLEO coll.), CLEO CONF 94-11.
- [14] K. Terasaki, Nuovo Cim. 66 A (1981) 475.
- [15] Particle Data Group, Phys. Rev. D 50 (1994) 1

V	$\Gamma_{V \rightarrow e^+ e^-}$ (GeV) [15]	f_V (GeV)
$\rho^0(0.770)$	6.74×10^{-6}	0.216
$\omega(0.782)$	0.60×10^{-6}	0.194
$\psi(3.097)$	5.27×10^{-6}	0.406
$\psi(3.685)$	2.44×10^{-6}	0.301
$\psi(3.770)$	2.83×10^{-7}	0.104
$\psi(4.040)$	7.28×10^{-7}	0.172
$\psi(4.160)$	0.77×10^{-6}	0.180
$\psi(4.415)$	0.47×10^{-6}	0.145

Table 1: Values of the measured decay rate of the vector meson V into $e^+ e^-$ pair and of the decay constant f_V . The V decay constant is defined by $\langle 0 | J_\mu^{em} | V \rangle \equiv e_V m_V f_V(m_V^2) \epsilon_\mu^V$ and is calculated from the experimental data for $\Gamma(V \rightarrow e^+ e^-)$, according to the formula
$$\Gamma(V \rightarrow e^+ e^-) = \frac{4\pi\alpha^2}{3} \frac{e_V^2 f_V^2(m_V^2)}{m_V} \left[1 - 4 \frac{m_e^2}{m_V^2} \right]^{1/2} \left[1 + 2 \frac{m_e^2}{m_V^2} \right].$$

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